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THE CHOICE OF FIDUCIAL POINTS IN DETERMINING PLATE CONSTANTS FOR LUNAR PHOTOGRAPHS

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Introduction

We wish to determine the mapping between an x-y coordinate grid superposed on a photograph of a section of the lunar surface and the direction cosines ξ and η of features in the photograph. This is conveniently done by using the method of Belsky (Belsky, 1962), in which ξ and η are found to be functions of a polynomial in x and y. Thus

$$\xi = A_1x^2 + B_1xy + C_1y^2 + D_1x + E_1y + F_1 \qquad (1)$$

and

$$\eta = A_2x^2 + B_2xy + C_2y^2 + D_2x + E_2y + F_2 \qquad (2)$$

The constants A,...,F in (1) and (2) are empirically determined. Sets of points x,y for which the direction cosines ξ , η are known are used to form a (redundant) set of simultaneous linear equations in A,...,F and this set is solved by the method of least squares. The points for which $\{(x,y), (\xi, \eta)\}$ are used in determining the con-

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stants A,...,F are the fiducial points.

Choice of Fiducial Points

A minimum of 20 sets $\{(x,y), (\xi, \eta)\}$ are used in determining the constants A,...,F. This assures that random errors in the measurements in individual $\{(x,y), (\xi, \eta)\}$ will not affect the A,...,F. Systematic errors, however, can result in a systematic error in the mapping of x and y to ξ and η.

In early determinations of the A,...,F for the first two sections of the Boston University Catalog of Lunar Craters (Hawkins and Mitchell, 1962, 1963 a) the sets $\{(x,y) (\xi, \eta)\}$ were taken from the Orthographic Atlas of the Moon (Arthur and Whitaker, 1960). This atlas contained duplicate photographs of the Photographic Lunar Atlas (Kuiper, 1960) with grids of ξ and η overprinted on the photographs. Sets of $\{(x,y), (\xi, \eta)\}$ could be quite conveniently obtained by measuring the ξ, η coordinates of given craters from the grid lines.

The two catalogs were prepared with an overlapping strip, and it became apparent that systematic errors were being introduced into the determination of A,...,F. Comparing the coordinates of particular craters which are in the overlapping strip, it was discovered that the ξ, η values of the same crater differed significantly between the two catalogs. The average deviation in ξ was .0003, considered acceptable, while the average deviation in η was .0008, amounting to about 1.33 km. Measurements on these photographs

can generally be made to accuracies within 1 km.

Analysis of the least squares determination of A,...,F shows that random errors in determination of the sets $\{(x,y), (\xi, \eta)\}$ should produce deviations of slightly less than .0003 in either ξ or η between catalogs. The further deviation must be due to a systematic error in the ξ, η coordinate grids of the Orthographic Lunar Atlas.

To remove the discrepancy caused by the systematic error, another method of determining $\{(x,y), (\xi, \eta)\}$ for fiducial points was used. Distinct craters with well measured x,y coordinates were identified, and their ξ, η coordinates obtained from the Consolidated Catalog of Selenographic Positions (Arthur, 1962). The ξ, η values thus used are those derived from direct measurement, mainly by S. A. Saunder, and represent the best values currently available.

When a new set A,...,F is then prepared with these fiducial points, a considerable reduction in the deviation of crater coordinates between catalogs is achieved. The deviation is now within the limits set by random error of measurement. The mean deviations are compared in Table I.

Table I

	$\Delta\xi$	$\Delta\eta$	ΔK
Coordinate Grid Fiducial Points	.0003	.0008	1.33
Consolidated Catalog Fiducial Points	.0003	.0003	.61

$\Delta\xi$ is the mean deviation of the ξ coordinate between Catalogs I & II (Hawkins & Mitchell 1962 and 1963a,b,c).

$\Delta\eta$ is the mean deviation of the η coordinate.

ΔK is the distance, in kilometers, corresponding to $\Delta\eta$.

The values in Table I are based on comparisons of 80 craters located in the overlapping strip between photographs C5a and C6b in the Photographic Lunar Atlas.

To obtain constants A,...,F allowing mapping between x,y and ξ,η to accuracies limited by random measuring errors, it is necessary to obtain fiducial points from the tabulation in the Consolidated Catalog of Selenographic Positions.

Correction of Systematic Errors

Catalogs of Lunar Craters I,II have been revised with corrected fiducial points, and have been republished as Revised Catalogs of Lunar Craters I,II (Hawkins and Mitchell 1963b,c). The further catalogs in the series of Boston University Survey of Lunar Craters have been prepared in accordance with the new method of determining fiducial points.

In many applications, such as computer programs, a mapping relationship such as (1) and (2) is more useful than the coordinate grid photographs. The mapping relationship has the added advantage of avoiding the systematic errors of the ξ, η grids when it is determined by the method described here and in Belsky (1962).

Table II contains the constants A,...,F in (1) and (2) as determined for five photographs in the Photographic Lunar Atlas.

Table II

	C5a	C6a	C7a	D4a	D5a
A_1	-1.2194×10^{-6}	-9.1634×10^{-6}	4.7668×10^{-6}	-8.1484×10^{-6}	-5.0832×10^{-6}
B_1	1.5425×10^{-6}	5.5938×10^{-6}	-2.0229×10^{-6}	-3.8399×10^{-7}	9.3448×10^{-7}
C_1	2.3433×10^{-6}	-9.1993×10^{-6}	4.1511×10^{-6}	-1.4123×10^{-5}	-1.5246×10^{-5}
D_1	-2.0248×10^{-2}	-1.8914×10^{-2}	-2.0627×10^{-2}	-1.9561×10^{-2}	-1.9696×10^{-2}
E_1	-2.2034×10^{-3}	-1.2718×10^{-3}	1.3028×10^{-3}	2.0771×10^{-4}	-1.3475×10^{-3}
F_1	3.2868×10^{-1}	3.6545×10^{-1}	3.3597×10^{-1}	3.4865×10^{-2}	6.2984×10^{-2}
A_2	1.3565×10^{-5}	-1.6389×10^{-6}	1.7207×10^{-5}	-7.0180×10^{-6}	-9.9877×10^{-6}
B_2	4.3432×10^{-7}	6.4253×10^{-6}	-9.2445×10^{-6}	2.3561×10^{-6}	4.1994×10^{-6}
C_2	1.7351×10^{-4}	-5.6524×10^{-6}	3.4549×10^{-5}	-1.2669×10^{-5}	-1.6785×10^{-6}
D_2	1.7230×10^{-3}	1.1026×10^{-3}	-1.5562×10^{-3}	5.9007×10^{-5}	1.4256×10^{-3}
E_2	-2.0035×10^{-2}	-1.9767×10^{-2}	-1.9455×10^{-2}	-1.9718×10^{-2}	-1.9495×10^{-2}
F_2	-4.3755×10^{-3}	-2.4702×10^{-1}	-4.9581×10^{-1}	2.8404×10^{-1}	-2.4401×10^{-4}

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